# GONDWANA UNIVERSITY GADCHIROLI

# **Proposed Syllabus For**

# **M.Sc.** Mathematics

# **Semester-I and Semester-II**

# **Under Choice Based Credit System**

# (C.B.C.S.)

# With effect from

# Academic Year: 2020-21

(Considered and approved by B.O.S. in Mathematics)

## M.Sc. Mathematics Semester wise Syllabus

## M.Sc. Semester- I

#### **CORE COURSES**

PSCMTH01: Group Theory & Ring Theory

PSCMTH02: Real Analysis

PSCMTH03: Topology

PSCMTH04: Linear Algebra

## **PSCMTH05: CORE ELECTIVE COURSE (Opt any one of the following)**

- (a) Numerical Analysis
- (b) Ordinary Differential Equations
- (c) Calculus of Variations
- (d) Number Theory
- (e) Fuzzy Mathematics- I

## M.Sc. Semester- II

### **CORE COURSES**

PSCMTH06: Field Theory

PSCMTH07: Lebesgue Measure Theory

PSCMTH08: Advanced Topics in Topology

PSCMTH09: Classical Mechanics

#### PSCMTH10: CORE ELECTIVE COURSE (Opt any one of the following)

- (a) Differential Geometry
- (b) Coding Theory
- (c) Cryptography
- (d) SCILAB Programming
- (e) Fuzzy Mathematics- II

Semester I for M.Sc. Program in Mathematics										
Code	Teaching Scheme (Hours/Week)		Credits			Examination Scheme				
	Theory	Total	Theory	Internal Total assessm ent		Duration in Hrs.	Maximum Marks		Total Marks	Minimum passing marks
							External assessment	Internal assessment		External assessment + Internal assessment
PSCMTH01	5	5	4	1	5	3	100	25	125	50
PSCMTH02	5	5	4	1	5	3	100	25	125	50
PSCMTH03	5	5	4	1	5	3	100	25	125	50
PSCMTH04	5	5	4	1	5	3	100	25	125	50
ELECTIVE COURSE PSCMTH05	5	5	4	1	5	3	100	25	125	50
	25	25	20	5	25		500	125	625	250

Semester II for M.Sc. Program in Mathematics										
Code	Teaching Scheme (Hours/Week)		Credits			Examination Scheme				
	Theory	Total	Theory	Internal assessm ent	Total	Duration in Hrs.	Maximum Marks		Total Marks	Minimum passing marks
							External assessment	Internal assessment		External assessment + Internal assessment
PSCMTH06	5	5	4	1	5	3	100	25	125	50
PSCMTH07	5	5	4	1	5	3	100	25	125	50
PSCMTH08	5	5	4	1	5	3	100	25	125	50
PSCMTH09	5	5	4	1	5	3	100	25	125	50
ELECTIVE COURSE PSCMTH10	5	5	4	1	5	3	100	25	125	50
	25	25	20	5	25		500	125	625	250

#### **Guidelines about Internal Assessment for all Semesters:**

The internal assessment marks shall be awarded by the concerned teacher. The internal assessment marks shall be sent to the University.

In case, the candidate fails in Theory Examination, the Internal Assessment marks will be carried forward for his next supplementary Examination.

There shall be no separate / extra allotment of work load to the teacher concerned. He/ She shall conduct the internal assessment activity during the regular teaching days / periods as a part of regular teaching activity.

The concerned teacher / department / college shall have to keep the record of all the internal assessment activities until six months after the declaration of the results of that semester.

Sr. No.	Activities	Max. Marks					
1	Attendance	05(Compulsory)					
Any Two of the Following Activities							
1	Seminar	10					
2	Unit Tests	10					
3	Home Assignments	10					

DISTRIBUTION OF MARKS FOR INTERNAL ASSESSMENT

Total Marks -25

Minimum Passing Marks -10

## **University Question Paper Pattern**

A student of M. Sc. Sem-I, Sem-II, Sem-III and Sem-IV in Mathematics has to attempt all five questions in each paper.

Q1 to Q4 are long answer questions with internal choice within unit whereas Q5 is compulsory question of short answers on all four units. Setting of the question paper is as under:

Total Marks: 100			Time 3 hours						
	(A)	Unit I	(10 marks)						
Q1	(B)	Unit I	(10marks)						
	OR								
	(C)	Unit I	(10 marks)						
	(D)	Unit I	(10marks)						
Q2	(A)	Unit II	(10 marks)						
	(B)	Unit II (10marks)							
	OR								
	(C)	Unit II	(10 marks)						
	(D)	Unit II	(10marks)						
	(A)	Unit III	(10 marks)						
	(B)	Unit III	(10marks)						
Q3	OR								
	(C)	Unit III	(10 marks)						
	(D)	Unit III	(10marks)						
Q4	(A)	Unit IV	(10 marks)						
	(B) Unit IV		(10marks)						
	OR								
	(C)	Unit IV	(10 marks)						
	(D)	Unit IV	(10marks)						
Q5	(A)	Unit I	(05 marks)						
	(B)	Unit II	(05 marks)						
	(C)	Unit III	(05 marks)						
	(D)	Unit IV	(05 marks)						

## **SEMESTER-I**

#### **Core Course Code: – PSCMTH01**

Credit - 05

### **Group Theory & Ring Theory**

#### UNIT-I

Permutation Groups, Normal subgroups and quotient groups, Isomorphism Theorems, Automorphisms, Conjugacy and G- Sets.

#### UNIT-II

Normal Series, Solvable groups, Nilpotent groups, Cyclic decomposition, Alternating groups  $A_{n,}$ , Simplicity of  $A_n$ .

#### UNIT-III

Direct products, Sylow theorems, Groups of order p<sup>2</sup> and pq.

#### UNIT-IV

Ideals, Homomorphisms, Sum and direct sum of ideals, Maximal and prime ideals, Nilpotent and Nil ideals, Zorn's Lemma.

#### **Text Book:**

Basic Abstract Algebra (Second Edition): P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Cambridge University Press.

- 1. Topics in Algebra (Second Edition): I. N. Herstein, John Wiley.
- 2. Abstract Algebra: David Summit and Richard M. Foote, John Wiley.

#### **Core Course Code: - PSCMTH02**

## **Real Analysis**

#### UNIT-I

Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Equicontinuous families of functions, The Stone-Weierstrass theorem.

#### UNIT-II

Differentiation, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem,

#### UNIT-III

Topological Manifolds, Compatible Charts, Smooth Manifolds, Examples of Smooth Manifolds.

#### UNIT-IV

Smooth functions on a Manifold, Smooth Maps between Manifolds, Diffeomorphisms, Smoothness in terms of Components, Examples on Smooth maps, Partial derivatives, The inverse function theorem.

#### **Text Books:**

- 1. Principles of Mathematical Analysis (Third Edition): Walter Rudin, McGRAW HILL Book Company.
- 2. An Introduction to Manifolds (Second Edition): Loring W. Tu, Springer.

#### **Reference Books:**

1. Methods of Real Analysis: R.R. Goldberg, John Wiley.

2. Calculus of Several Variables: C Goffman, Harper and Row.

## Topology

## UNIT-I

Equipotent sets, Cardinal Numbers, Order Types, Ordinal Numbers.

## UNIT-II

Open sets and limit points, Closed sets and closure, Operators and Neighbourhoods, Bases and Relative topologies.

## UNIT-III

Connected sets and components, Compact and countably compact spaces, Continuous functions, Homeomorphisms.

## UNIT-IV

T<sub>0</sub>- and T<sub>1</sub>- spaces, T<sub>2</sub>- spaces and sequences, Axioms of countability, Separability.,Regular and Normal spaces.

### Text Book:

Foundations of General Topology: W. J. Pervin, Academic press, 1964.

- 1. Topology (Second edition): J. R. Munkres, Prentice Hall of India, 2002.
- 2. Introduction to Topology and Modern Analysis: G. F. Simmons, Mc Graw Hill 1963.
- 3. General Topology: J.L. Kelley, Van Nostrand, 1995.
- 4. Introduction to general Topology: K.D.Joshi, Wiley Eastern Ltd., 1983.

## Linear Algebra

### UNIT-I

Vector spaces: Introduction, Vector spaces, subspaces, Linear combinations and system of linear equations, linear dependence and independence, Bases and dimension, Maximal Linear Independent Subsets.

#### UNIT-II

Linear Transformations and Matrices: Linear Transformations, Null spaces, and ranges, The matrix representation of a linear transformation, Composition of linear transformations, Invertibility and Isomorphisms, The change of Coordinate matrix, Dual spaces, and Homogeneous Linear Differential equations with constant coefficients.

#### UNIT-III

Diagonalization: Eigen values and eigen vectors, Diagonalizability, Invariant Subspaces and the Cayley-Hamilton Theorem.

#### UNIT-IV

Inner Product Spaces: Inner products and Norms, The Gram-Schmidt Orthogonalization process and orthogonal complements, the adjoint of a linear operator, Normal and self-adjoint operators, Unitary and orthogonal operators and their matrices, orthogonal projections and the spectral theorem, Quadratic forms. Jordan Canonical form I, Jordan Canonical form II, The minimal polynomial, Rational Canonical form.

#### Text Book:

Linear Algebra (Fourth Edition): S. H. Friedberg, A. J. Insel and L. E. Spence, Prentice-Hall International Inc.

**Scope:** Ch 1: Art.1.1 to 1.7, Ch 2: Art. 2.1 to 2.7, Ch 5: Art 5.1,5.2,5.4, Ch 6: Art 6.1 to 6.7, Ch 7: Art 7.1 to 7.4.

- 1. Linear Algebra (Second Edition): Vivek Sahai, Vikas Bist, Narosa Publishing House
- 2. Topics in Algebra: I. N. Herstein, Macmillan, Indian Edition.
- 3. Introduction to Linear Algebra (Second Edition): S. Lang, Springer International Edition.
- 4. Linear Algebra: K. Hoffman, R. Kunze, Prentice Hall of India.
- 5. Linear Algebra (Second Edition): J. H. Kwak, S. Hong Birkhauser Verlag.
- 6. Linear Algebra: A pure Mathematical Approach: Harvey E.Rose, Birkhäuser Verlag.

## (a) Numerical Analysis

## UNIT-I

The Bisection method, Newton's method, The Secant method, Muller's method, A General theory for one point iteration methods, Aitken extrapolation for linearly convergent sequences, The Numerical evaluation of multiple roots, Roots of polynomials, Systems of Non-linear equations, Newton's method for non-linear systems.

## UNIT-II

Polynomial interpolation theory, Newton divided differences, Finite difference and table oriented interpolation formulas, Errors in data and Forward-differences, Hermite interpolation.

#### UNIT-III

The Weierstrass theorem and Taylor's theorem, The minimax approximation problem, The least square approximation problem, Orthogonal polynomial, The least square approximation problem (continued), Minimax approximation.

#### UNIT-IV

The Trapezoidal rule and Simpson's rule, Newton- Cotes integration formulas, Gaussian Quadrature.

#### Text book:

An Introduction to Numerical Analysis: Kendal E. Atkinson, John Wiley and sons, Inc.

- 1. Numerical Methods: S. R. K. Iyengar, R. K. Jain, New Age International Publishers.
- 2. Introductory Methods of Numerical analysis (Fifth Edition): S. S. Sastry, Prentice Hall India Pvt. Ltd..

## (b) Ordinary Differential Equations

## UNIT-I

Linear equations of first order, The equation y' + ay = 0, The equation y' + ay = b(x), The general linear equation of first order, The second order homogeneous equation, Initial value problems, Linear dependence and independence, A formula for the Wronskian, The homogeneous equation of order n, The non-homogeneous equation of order n, A special method for solving non-homogeneous equation.

## UNIT-II

Linear equations with variable coefficients, Initial value problem for the homogeneous equation, Solution of the homogeneous equation, Wronskian and linear independence, Reduction of order of a homogeneous equation, Non-homogeneous equation, Homogeneous equations with analytic coefficients, The Legendre equation, The Euler equation, Second order equation with regular singular points, The Bessel equation.

## UNIT-III

Equations with variables separated, Exact equation, The method of successive approximations, The Lipschitz condition, Convergence of successive approximations, Nonlocal existence of solutions, Approximations to and uniqueness of the solutions.

## UNIT-IV

Some special equations, Complex n-dimensional space, Systems as vector equations, Existence and uniqueness of solutions to systems, Existence and uniqueness for linear systems, Equations of order n.

## Text Book:

An Introduction to Ordinary Differential Equation: E. A. Coddington, Prentice-Hall of India Pvt. Ltd., New Delhi.

Scope: Chapter 1 to 6.

- Differential Equations with Applications and Historical Notes (Second edition): G. F. Simmons, McGraw Hill Book Co.
- 2. Ordinary Differential Equations: G. Birkhoff and G. C. Rota, John Wiley and Sons.
- 3. Ordinary Differential Equations A First Course: D. Somasundaram,, Narosa Publishing House.

## (c) Calculus of Variations

## UNIT-I

Functionals- some simple variational problems, The variation of a functional, A necessary condition for an extremum, The simplest variational problem Euler's equation, The case of several variables, A simple variable end point problem, The variational derivative, Invariance of Euler's equation.

## UNIT-II

The fixed end point problem for n-unknown functions, Variational problem in parametric form, Functionals depending on higher order derivatives, Variational problems with subsidiary conditions.

## UNIT-III

Derivation of the basic formula, End points lying on two given curves or surfaces, Broken extremals, The Weierstrass Erdmann conditions.

### UNIT-IV

The canonical form of Euler equations, First integrals of the Euler equations, The Legendre transformation, Canonical transformations, Noether's Theorem, The principle of least action, Conservation laws, The Hamilton Jacobi equation, Jacobi theorem.

#### **Text Book:**

Calculus of Variations: I. M. Gelfand and S. V. Fomin, Dover Publications, 2000.

#### **Reference Books:**

1. Calculus of Variations with Applications: A. S. Gupta, Prentice-Hall of India, 2008.

2. Problems and Exercises in the Calculus of Variations: M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, Mir Publishers, Moscow, 1975.

## (d) Number Theory

## UNIT-I

Congruence, Solutions of Congruence, Chinese Remainder Theorem, Techniques of Numerical Calculation, Public-Key Cryptography.

## UNIT-II

Prime Power Moduli, Prime Modulus, Primitive Roots and Power Residues, Congruence of Degree Two, Prime Modulus.

## UNIT-III

Quadratic Residues, Quadratic Reciprocity, Greatest Integer Function.

## UNIT-IV

Arithmetic Functions, Mobius Inversion Formula, The equation ax + by = c, Simultaneous linear equations, Pythagorean Triangles, Assorted Examples.

#### **Text Book:**

An Introduction to the Theory of Numbers (Fifth edition): Ivan Niven, Herbert S. Zuckerman, Hugh L. Montgomery John Wiley & Sons Inc. 1991.

- 1. Elementary Number Theory (Sixth Edition): David M. Burton, McGraw-Hill Higher Edition,.
- 2. An Introduction to Number Theory: Graham Everest, Thomas Ward, Springer, 2005.

## (e) Fuzzy Mathematics - I

## UNIT-I

From classical (Crisp) Sets to Fuzzy Sets. Fuzzy sets versus Crisp sets, Operations on Fuzzy sets.

#### UNIT-II

Fuzzy Arithmetic.

### UNIT-III

Fuzzy relations.

#### UNIT-IV

Fuzzy relation equations.

#### **Text Book:**

Fuzzy Sets and Fuzzy Logic, Theory and Applications. George J. Klir and Bo Yuan, Prentice Hall India.

- 1. Fuzzy Logic with Engineering Applications (Third Edition): Timothy J. Ross, Wiley, 2010.
- 2. Fuzzy Sets and systems Theory and Applications: Didier Dubois, Henri Prade, Academic Press, 1980.

## **SEMESTER-II**

#### **Core Course Code:- PSCMTH06**

Credit - 05

## **Field Theory**

#### UNIT-I

Unique factorization domains, Principal Ideal domains, Euclidean domains, Polynomial rings over unique factorization domains.

#### UNIT-II

Irreducible polynomials and Eisenstein criterion, Adjunction of roots, Algebraic extensions, Algebraically closed fields.

#### UNIT-III

Splitting fields, Normal extensions, Multiple roots, Finite fields, Separable extensions.

#### UNIT-IV

Automorphism groups and fixed fields, Fundamental theorem of Galois theory, Fundamental theorem of algebra.

#### **Text Book:**

Basic Abstract Algebra (Second Edition): P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Cambridge University Press.

- 1. Topics in Algebra, I. N. Herstein, Second Edition, John Wiley.
- 2. Abstract Algebra, David S. Dummit and Richard M. Foote, John Wiley.

## Lebesgue Measure Theory

## UNIT-I

Introduction, Outer measure, Measurable sets and Lebesgue measure, A nonmeasurable set, Measurable functions, Littlewood's three principles.

## UNIT-II

The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, Integral of a non-negative function, General Lebesgue integral, Convergence in measure.

## UNIT-III

Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity, Convex functions.

## UNIT-IV

The  $L^P$ - Spaces, The Holder and Minkowski inequalities, Convergence and Completeness, Bounded linear functionals on  $L^P$ - Spaces.

#### **Text Book:**

Real Analysis (Third edition): H. L. Royden, Prentice Hall India, 1988.

#### **Reference Books:**

1. Measure theory and Integration: De Barra G., Wiley Eastern Limited, 1981.

2. An introduction to Measure & Integration: Inder K. Rana, Narosa Publishing House.

#### **Core Course Code:- PSCMTH08**

## **Advanced Topics in Topology**

## UNIT-I

Completely normal spaces, Completely Regular spaces, Metric spaces as Topological spaces, Topological Properties.

### UNIT-II

Finite products, Product invariant properties, Metric products, Tichonov topology, Tichonov theorem.

### UNIT-III

Quotient topology, Urysohn's metrization theorem, Paracompact spaces.

#### UNIT-IV

Nets and filters.

#### Text books:

- 1. Foundations of General Topology: W. J. Pervin, Academic press, 1964. (For Unit-I, II & III)
- 2. Introduction to General Topology: K. D. Joshi, Wiley Eastern Ltd. 1983. (For Unit -IV)

- 1. Topology (Second Edition): J. R. Munkres, Prentice Hall of India, 2002.
- 2. Introduction to topology and modern analysis: G. F. Simmons, Mc Graw Hill 1963.
- 3. General Topology: J. L. Kelley, Van Nostrand, 1995.

## **Classical Mechanics**

## UNIT-I

Hamilton's Principle, Some techniques of calculus of variations, Derivation of Lagrange equations from Hamilton's principle, Extension of principle to nonholonomic systems, Conservation theorems and symmetry properties.

### UNIT-II

Legendre transformations and the Hamilton equations of motion, Cyclic coordinates and conservation theorems, Routh's procedure and oscillations about steady motion, The Hamiltonian formulation of relativistic mechanics, The Principle of least action.

#### UNIT-III

The equations of canonical transformation, Examples of canonical transformations, The symplectic approach to canonical transformations, Poisson brackets and other canonical invariants.

#### UNIT-IV

Equations of motion, Infinitesimal canonical transformations, and conservation theorems in the Poisson bracket formulation, The angular momentum Poisson bracket relations, Symmetry groups of mechanical systems, Liouville's theorem.

#### Text Book:

Classical Mechanics (Second Edition): H. Goldstein, Narosa publishing house, New Delhi.

- 1. Lectures in Analytic Mechanics: F. Gantmacher, MIR Publishers, Moscow, 1975.
- 2. Classical Mechanics: Narayan Chandra Rana and Pramod Sharad Chandra Jog, Tata Mc Graw Hill.

#### **Core Course Code:- PSCMTH10**

## (a) Differential Geometry

### UNIT-I

The First Fundamental Form and Local Intrinsic Properties of a Surface.

## UNIT-II

Godesics on a Surface.

## UNIT-III

The Second Fundamental Form and Local Non-Intrinsic Properties of a Surface.

## UNIT-IV

The Fundamental Equations of Surface Theory.

### **Text Book:**

Differential Geometry A First Course: D. Somasundaram, Eighth Reprint 2017, Narosa Publishing House. Scope: Chapter 2,3,4 &5.

- 1. Geometry of curves and surfaces: M. P. Do Carmo, Academic Press.
- 2. An introduction to Differential Geometry: T. J. Wilmore, Oxford University Press.

## (b) Coding Theory

## UNIT-I

Error detection, correction and decoding: Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbour / minimum distance decoding, Distance of a code.

#### UNIT-II

Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cosets, Nearest neighbour decoding for linear codes, Syndrome decoding.

#### UNIT-III

Cyclic codes: Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes.

#### UNIT-IV

Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Decoding of BCH codes.

#### Text Book:

Coding Theory - A First Course (First edition): San Ling and Chaoping Xing, Cambridge University Press. Scope: Chapters 2,4,7,8.

# Reference Book:

A Student's Guide to Coding and Information Theory: Stefan M. Moser, Po - Ning Chen, Cambridge University Press, 2012.

## (c) Cryptography

## UNIT-I

Time estimates for doing arithmetic, Divisibility and Euclidean algorithm, Congruences, Some applications to factoring, Finite fields, Quadratic residues and reciprocity.

## UNIT-II

The idea of public key cryptography, RSA, Pseudo primes, The Rho method.

## UNIT-III

The ElGamal cryptosystem, Algorithms for discrete logarithm problem, Security of ElGamal systems, The ElGamal signature scheme.

## UNIT-IV

Elliptic curves: Basic Facts, Elliptic curve cryptosystems, Elliptic curve primality test, Elliptic curve factorization.

### **Text Books:**

1. A Course in Number Theory and Cryptography (Second edition): Neal Koblitz, Springer Verlag.

2. Cryptography: Theory and practice (Third Edition): Douglas R. Stinson, CRC Press.

Scope:

Unit - I - From Koblitz's book (Chapter 1 and Chapter 2 excluding Existence and uniqueness of finite fields with prime power number of elements)

Unit - II – From Koblitz's book (Chapter 4 –sections 1 and 2, Chapter 5- sections 1 and 2)

Unit - III – From Stinson's book (Chapter 6- section 1, 2 and 7, Chapter 7- section 3)

Unit - IV - From Koblitz's book (Chapter 6)

## (d) SCILAB Programming

#### UNIT-I

Introduction to SCILAB, The SCILAB Environment, Scalars & vectors.

#### **UNIT-II**

Matrices, Programming in SCILAB.

#### UNIT-III

Polynomials, Menus and Dialogue Boxes, Graphic Output.

#### **UNIT-IV**

String handling functions, Statistics.

#### **Text Book:**

SCILAB (A free software to MATLAB) (First Edition): Er. Hema Ramchandran and Achuthsankar S. Nair, S. Chand & Company Ltd., India, 2012.

## (e) Fuzzy Mathematics-II

## Unit-I:

Possibility theory

## Unit-II:

Fuzzy Logic

## Unit-III:

Constructing Fuzzy sets and operations on Fuzzy sets, Approximate reasoning.

## Unit-IV:

Fuzzy Systems, Pattern Recognition.

#### **Text Book:**

Fuzzy Sets and Fuzzy Logic, theory and applications. George J. Klir and Bo Yuan, Prentice Hall India.

- 1. Fuzzy Logic with Engineering Applications (Third Edition): Timothy J. Ross, Wiley, 2010.
- 2. Fuzzy Sets and systems Theory and Applications: Didier Dubois, Henri Prade, Academic Press, 1980.